# The University of Texas at Austin 

Dept. of Electrical and Computer Engineering
Midterm \#2

Date: November 16, 2021
Course: EE 313 Evans

Name: $\qquad$
Last,
First

- This in-person exam is scheduled to last 75 minutes.
- Open books, open notes, and open class materials, including homework assignments and solution sets and previous midterm exams and solutions.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network.
- Please disable all wireless connections on your calculator(s) and computer system(s).
- Please mute all computer systems.
- Please turn off all phones.
- No headphones are allowed.
- All work should be performed on the midterm exam. If more space is needed, then use the backs of the pages.
- Fully justify your answers. If you decide to quote text from a source, please give the quote, page number and source citation.

| Problem | Point Value | Your score | Topic |
| :---: | :---: | :---: | :---: |
| 1 | 27 |  | System Properties |
| 2 | 27 |  | Convolution |
| 3 | 26 |  | Highpass Filters |
| 4 | 20 |  | Wireless Localization |
| Total | 100 |  |  |

Problem 2.1. System Properties. 27 points.
Each discrete-time system has input $x[n]$ and output $y[n]$, and $x[n]$ and $y[n]$ might be complex-valued.
Each continuous-time system has input $x(t)$ and output $y(t)$, and $x(t)$ and $y(t)$ might be complex-valued.
Determine if each system is linear or nonlinear, time-invariant or time-varying, and stable or unstable.
You must either prove that the system property holds in the case of linearity, time-invariance, or stability, or provide a counter-example that the property does not hold. Providing an answer without any justification will earn 0 points.
$\left.\begin{array}{|c|c|c|l|l|}\hline \text { Part } & \begin{array}{c}\text { System } \\ \text { Name }\end{array} & \text { System Formula } & \text { Linear? } & \begin{array}{c}\text { Time- } \\ \text { Invariant? }\end{array} \\ \hline \text { (a) } & \begin{array}{c}\text { Averaging } \\ \text { Filter }\end{array} & \begin{array}{c}y[n]=x[n]+x[n-1] \\ \text { for } n \geq 0 \text { and } x[-1]=0\end{array} & & \\ \hline \text { (b) } & \begin{array}{c}\text { Frequency } \\ \text { modulation }\end{array} & y(t)=\cos \left(2 \pi f_{c} t+2 \pi k_{f} \int_{0}^{t} x(\lambda) d \lambda\right) \\ \text { for } t \geq 0 . f_{c} \text { and } k_{f} \text { are real-valued constants. }\end{array}\right)$
(a) Averaging filter: $y[n]=x[n]+x[n-1]$ for $n \geq 0$ and $x[-1]=0.9$ points.
(b) Frequency Modulation: $y(t)=\cos \left(2 \pi f_{c} t+2 \pi k_{f} \int_{0}^{t} x(\lambda) d \lambda\right)$ for $t \geq 0.9$ points.
(c) Time reversal: $y(t)=x(-t)$ for $-\infty<t<\infty$. 9 points.

Problem 2.2 Convolution. 27 points.
In continuous-time systems, an linear time-invariant (LTI) integrator is a building block. Its inputoutput relationship is

$$
y(t)=\int_{-\infty}^{t} x(\tau) d \tau
$$

The impulse response is $h(t)=u(t)$ where $u(t)$ is 1 for $t \geq 0$ and 0 otherwise.
The equivalent system in discrete time is the LTI running summation system. Its input-output relationship is

$$
y[n]=\sum_{m=-\infty}^{n} x[m]
$$

The impulse response is $h[n]=u[n]$ where $u[n]$ is 1 for $n \geq 0$ and 0 otherwise.
(a) Plot the output $y[n]$ of the LTI running summation when the input is a five-point rectangular pulse $x[n]=\delta[n]+$ $\delta[n-1]+\delta[n-2]+\delta[n-3]+\delta[n-4]$ ? On the right, $x[n]$ is plotted for $-2 \leq n \leq 6$. 9 points.

(b) Give a formula for the output $y[n]$ of the LTI running summation when the input is $x[n]=u[n]$. The formula should be valid for $-\infty<n<\infty .9$ points.
(c) Give a formula for the output $y(t)$ of the LTI integrator when the input is $x(t)=u(t)$. The formula should be valid for $-\infty<t<\infty$. 9 points.

Problem 2.3. Highpass Filters. 26 points.
A highpass filter passes high frequencies and attenuates low frequencies.
Shown below are the pole-zero plots for two discrete-time linear time-invariant (LTI) highpass filters.
(a) LTI first-order finite impulse response (FIR) filter.

Pole-zero plot on the right. Zero at $z=1$ and pole at $z=0$.
i. Write the transfer function $H(z)$ including the region of convergence. 6 points.

ii. Give a formula for the frequency response. 3 points.
iii. Plot the magnitude response. 4 points.
(b) LTI first-order infinite impulse response (IIR) filter.

Pole-zero plot on the right. Pole at $z=-0.9$ and zero at $z=0$.
i. Write the transfer function $H(z)$ including the region of convergence. 6 points.

ii. Give a formula for the frequency response. 3 points.
iii. Plot the magnitude response. 4 points.

Problem 2.4 Wireless Localization. 20 points.
Mini-Project \#2 used radar (electromagnetic waves and signal processing) to locate an object in an environment. Applications include automotive and 6 G cellular communication systems.
A block diagram for the radar system to determine the distance to the object follows.


The radar system estimates the round-trip time $T_{d}$ from the transmitter to the object and from the object to the receiver. Transmitter and receiver are in the same location. Distance $d$ can be computed using $d=\frac{1}{2} c T_{d}$ where $c$ is the speed of propagation of electromagnetic waves in air (speed of light).
(a) What are the advantages of using a chirp pulse for the transmitted waveform? 6 points.
(b) Describe the role of the matched filter in the receiver. Why is it called a matched filter? What is being matched? 6 points.
(c) How did the radar system use the chirp pulse and matched filter to estimate the round-trip time $T_{d}$ from the transmitter to the object and from the object to the receiver? 8 points.

